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Newton's Axiomatic Method

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Isaac Newton (1642–1727) was born in Woolsthorpe (Lincolnshire). His yeoman father died before Isaac's birth. Newton's mother remarried when he was three, and his upbringing was relegated largely to a grandmother, until the death of his stepfather in 1653.

Newton attended Trinity College, Cambridge, and received a BA degree in 1665. During 1665–7, Newton stayed at Woolsthorpe to avoid the plague. This was a period of immense creativity, in which Newton formulated the binomial theorem, developed the "method of fluxions" (calculus), constructed the first reflecting telescope, and came to realize the *universal* nature of gravitational attraction.

Newton was appointed Professor of Mathematics at Cambridge in 1669, and was elected a fellow of the Royal Society in 1672. Shortly thereafter, he communicated to the Society his findings on the refractive properties of light. An extended debate ensued with Robert Hooke and others. The controversy with Hooke deepened upon publication of the *Mathematical Principles of Natural Philosophy* (1687). Hooke complained that Newton had appropriated his position that planetary motions could be explained by a rectilinear inertial principle in combination with a $1/r^2$ force emanating from the sun. Newton replied that he had come to this conclusion before Hooke, and that only he could prove that a $1/r^2$ force law leads to elliptical planetary orbits.

Newton became Warden of the Mint in 1696 and displayed considerable talent for administration. He was elected President of the Royal Society in 1703, and from this vantage-point carried on a running feud with Leibniz over priorities in the development of the calculus. In 1704, Newton published the *Opticks*, a model of

experimental inquiry. He included in the "Queries" at the end of this book a statement of his view of scientific method.

Throughout his life Newton studied the Biblical records from the standpoint of a Unitarian commitment. Extensive notes on the chronology of ancient kingdoms and the exegesis of *Daniel* have been found among his papers.

The Method of Analysis and Synthesis

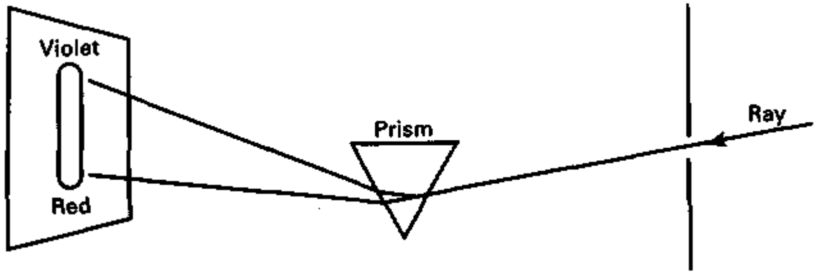
Newton's comments about scientific method were directed primarily against Descartes and his followers. Descartes had sought to derive basic physical laws from metaphysical principles. Newton opposed this method of theorizing about nature. He insisted that the natural philosopher base his generalizations on a careful examination of phenomena. Newton declared that "although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions, yet it is the best way of arguing which the Nature of Things admits of".[1]

Newton opposed the Cartesian method by affirming Aristotle's theory of scientific procedure. He referred to this inductive–deductive procedure as the "Method of Analysis and Synthesis". By insisting that scientific procedure should include both an inductive stage and a deductive stage, Newton affirmed a position that had been defended by Grosseteste and Roger Bacon in the thirteenth century, as well as by Galileo and Francis Bacon at the beginning of the seventeenth century.

Newton's discussion of the inductive–deductive procedure was superior to that of his predecessors in two respects. He consistently stressed the need of experimental confirmation of the consequences deduced by Synthesis, and he emphasized the value of deducing consequences that go beyond the original inductive evidence.

Newton's application of the Method of Analysis and Synthesis reached fruition in the investigations of the *Opticks*. For example, in a deservedly famous experiment, Newton passed a ray of sunlight through a prism such that an elongated spectrum of colour was produced on the far wall of a darkened room.

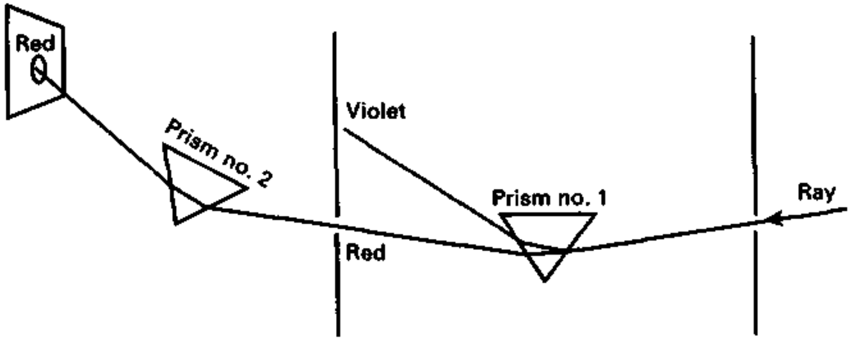
Newton applied the Method of Analysis to induce the explanatory principle that sunlight comprises rays of differing colours, and that each colour is refracted by the prism through a characteristic angle. This was not a simple inductive generalization on Newton's part. Newton did not affirm merely that all prisms under similar circumstances would produce spectra similar to those he had observed. His more important conclusion was about the nature of light itself, and it required an "inductive leap" to conclude that sunlight is made up



Newton's One-Prism Experiment

of rays which have different refractive properties. After all, other interpretations of the evidence are possible. Newton might have concluded, for instance, that sunlight is indivisible, and that the spectral colours are produced instead by some sort of secondary radiation within the prism.

Given the "theory" that sunlight does comprise rays of different colours and refractive properties, Newton then applied the Method of Synthesis to deduce certain further consequences of the theory. He noted that if his theory were correct, then passing light of a particular colour through a prism should result in a deflection of the beam through the angle characteristic of that colour, but no resolution of the beam into other colours. Newton confirmed this consequence of his theory of colours by passing light from one small band of the spectrum through a second prism.[2]



Newton's Two-Prism Experiment

Inductive Generalization and the Laws of Motion

Newton also claimed to have followed the Method of Analysis and Synthesis in his great work on dynamics, the *Mathematical Principles of Natural Philosophy* (1686). In this volume, he reported that he had formulated the three laws of motion upon application of the Method of Analysis. Newton declared that in experimental philosophy "particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered."[3]

Newton did not discuss the nature of the inductive process which proceeds from phenomena to particular propositions to the laws of motion. Whether or not it is correct to say that the laws of motion were discovered upon application of the Method of Analysis depends on how broadly one construes "induction".

Aristotle, for instance, admitted intuitive insight as a bona fide inductive method. Aristotle's theory of procedure thus could account for generalizations about weightless, infinitely rigid levers, ideal pendulums, and inertial motion. Indeed, it would be difficult to find a scientific interpretation whose origin could not be attributed to intuitive insight.

Most natural philosophers, however, have taken a more restricted view of induction, limiting it to a small number of techniques for generalizing the results of observation. These techniques include simple enumeration, and the methods of agreement and difference.

It is clear that Newton's Laws were not discovered upon application of these inductive techniques. Consider the first law. It specifies the behaviour of those bodies which are under the influence of no impressed forces. But no such bodies exist. And even if such a body did exist, we could have no knowledge of it. Observation of a body requires the presence of an observer or some recording apparatus. But on Newton's own view, every body in the universe exerts a gravitational attractive force on every other body. An observed body cannot be free of impressed forces. Consequently, the law of inertia is not a generalization about the observed motions of particular bodies. It is, rather, an abstraction from such motions.

Absolute Space and Absolute Time

Moreover, Newton maintained that the three laws of motion specify how bodies move in Absolute Space and Absolute Time. This is a further abstraction on Newton's part. Newton contrasted Absolute Space and Time with their "sensible measures" which are determined experimentally.

Newton's distinction between the "true motions" of bodies in Absolute Space and Time and the "sensible measures" of these motions has a Platonic ring that suggests a dichotomy of reality and appearance. On Newton's view, Absolute Space and Absolute Time are ontologically prior to individual substances and their interactions. He believed, moreover, that an understanding of sensible motions can be achieved in terms of true motions in Absolute Space.

Newton recognized that to establish that a sensible measure of a body's motion is its true motion, or that a sensible motion is related in some specific way to its true motion, it would be necessary to specify both Absolute temporal intervals and coordinates in Absolute Space. But he was not certain that these requirements can be met.

With respect to Absolute Time, Newton declared that "it may be, that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the flowing of absolute time is not liable to any change."[4] However, Newton did indicate that some sensible measures of time are preferable to others. He suggested that for the definition of temporal intervals, the eclipses of Jupiter's moons and the vibrations of pendulums are superior to the apparent motion of the sun around the Earth.[5]

But even if Absolute Time could be measured, it still would be necessary to locate a body in Absolute Space before its absolute motion could be determined. Newton was convinced that Absolute Space must exist, and he advanced both theological arguments and physical arguments for its existence, but he was less certain that bodies could be located in this space.

Newton maintained on theological grounds that since the universe was created *ex nihilo*, there must exist a receptacle within which created matter is distributed. He suggested that Absolute Space is an "emanent effect" of the Creator, a "disposition of all being" which is neither an attribute of God nor a substance coeternal with God. Newton criticized Descartes's identification of extension and body as offering a path to atheism, since, according to Descartes, we can achieve a clear and distinct idea of extension independently of its nature as a creation of God.[6]

The most important of Newton's physical arguments for the existence of Absolute Space was his analysis of the motion of a rotating, water-filled bucket.* He noted that if such a bucket were suspended from a twisted rope and allowed to rotate as the rope unwinds, the water surface remains a plane for a time and only gradually assumes a concave shape. At length the water rotates at the same rate as the bucket. Newton's experiment showed that the deformation of the water surface could not be correlated with an acceleration of the water relative to the bucket, since the water surface is successively a plane and concave when there is a relative acceleration, and since the water surface may be either a plane or concave when there is no relative acceleration.

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* Many interpreters have taken Newton to have cited the bucket experiment as evidence for the existence of Absolute Space. Ronald Laymon has argued, however, that Newton described the rotating bucket merely to illustrate that absolute motions can be distinguished from relative motions on the prior assumption that Absolute Space does exist.[7]

Newton's Bucket Experiment

Event	*Acceleration of water relative to bucket in earth-centred co-ordinate system*	*Surface of water*
1. Bucket stationary	no	plane
2. Bucket released	yes	plane
3. At maximum rotation	no	concave
4. Bucket arrested	yes	concave
5. Water at rest	no	plane

Newton maintained that deformation of the water surface indicates that a force is acting. And the second law of motion associates force and acceleration. But this acceleration of the water is an acceleration with respect to what? Newton concluded that since the acceleration associated with deformation is not an acceleration relative to the bucket, it must be an acceleration with respect to Absolute Space.[8]

Subsequently, numerous writers have pointed out that Newton's conclusion does not follow from his experimental findings. Ernest Mach, for example, suggested that the deformation be correlated, not with an acceleration with respect to Absolute Space, but with an acceleration with respect to the fixed stars.[9]

However, even if Newton were correct to conclude that the bucket experiment demonstrates the existence of an absolute motion, this would not suffice to specify a system of co-ordinates for locating positions in Absolute Space. Newton conceded this. Moreover, he admitted that there may be no single body which is at rest with respect to Absolute Space, and which may serve as a reference point for measuring distances in this space.[10]

Newton thus admitted that it may not be possible to achieve a wholly satisfactory correspondence between observed motions and true motions in Absolute Space. His explicit discussion of this problem of correspondence indicates that he followed an axiomatic method in the *Principia* rather than the inductive method of Analysis.

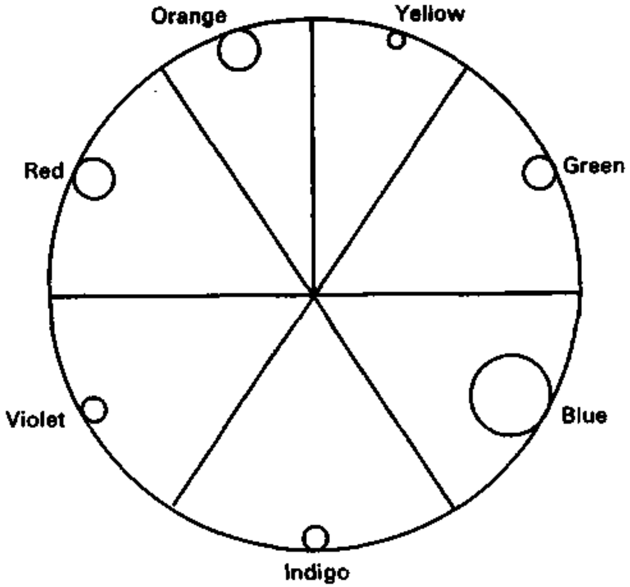
An Axiomatic Method

There are three stages in Newton's axiomatic method. The first stage is the formulation of an axiom system. On Newton's view, an axiom system is a deductively organized group of axioms, definitions, and theorems. Axioms are propositions that cannot be deduced from other propositions within the system, and theorems are the deductive consequences of these axioms. The three laws of motion are the axioms of Newton's theory of mechanics. They stipulate invariant relations among such terms as 'uniform motion in a right line', 'change of motion', 'impressed force', 'action', and 'reaction'. The axioms are:

I. Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.
II. The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
III. To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.[11]

Newton clearly distinguished the "absolute magnitudes" which appear in the axioms from their "sensible measures" which are determined experimentally. The axioms are *mathematical principles* of natural philosophy which describe the true motions of bodies in Absolute Space.

The second stage of the axiomatic method is to specify a procedure for correlating theorems of the axiom system with observations. Newton usually required that axiom systems be linked to events in the physical world.



Newton's Theory of Colour-Mixing

However, he did submit for consideration a Theory of Colour-Mixing in which the axiom system was not properly linked to experience.[12] Newton specified that a circle be drawn and be subdivided into seven wedges—one for each of the "principal colours" of the spectrum—such that the widths of the wedges are proportional to the musical intervals in the octave. He further specified that the "number of rays" of each colour in the mixture be represented by a circle of greater or smaller radius located at the midpoint of the arc for each colour present in the mixture. Newton indicated that the centre of gravity of these circles gives the resultant colour of the mixture.

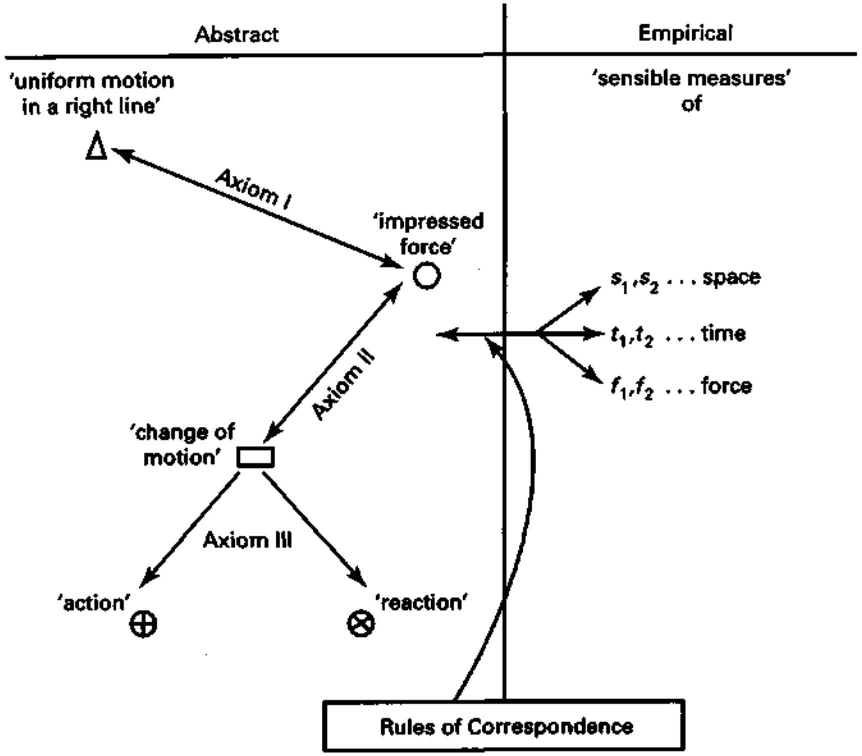
Newton's axiom on slicing the pie to satisfy musical harmonies is reminiscent of Kepler's Pythagorean speculations. The axiom certainly is not an inductive generalization. Nevertheless, even though there is no evidence in support of the pie-slicing axiom, the theory would be useful if the results of mixing colours could be calculated from it. But Newton failed to provide an empirical interpretation for the phrase "number of rays". Since he did not stipulate how the diameters of the circles are to be determined, Newton's theory of colour-mixing has no empirical significance.

Newton's mechanics, on the other hand, does have empirical significance. He did link his axiom system for mechanics to events in the physical world. He achieved the required link by selecting "Rules of Correspondence" for the conversion of statements about Absolute spatial and temporal intervals into statements about measured spatial and temporal intervals.

In the case of spatial intervals, Newton asserted as a "hypothesis" that the centre of gravity of the solar system is immovable, and therefore a suitable reference point for the determination of Absolute distances. He thus was able to apply his axiom system to actual motions by selecting a co-ordinate system the origin of which is the centre of gravity of the solar system.

I. Bernard Cohen has suggested that Newton meant by "hypothesis" in this context a proposition that he was unable to prove.[13] But although Newton was unable to prove that the centre of gravity of the solar system is immovable, his hypothesis is consistent with his interpretation of the bucket experiment. On this interpretation, the recession of water towards the walls of the bucket is an acceleration with respect to Absolute Space. According to Newton, this centrifugal acceleration typifies those effects which distinguish motions with respect to Absolute Space from merely relative motions.[14] Newton believed that "the motion which causes the Earth to endeavour to recede from the Sun" is likewise an Absolute Motion.[15] Since the centre of gravity of the solar system is the "centre" of this motion of revolution (at least in so far as the motion is approximately circular), Newton's hypothesis fits in with his views on Absolute Motion.

In the case of temporal intervals, Newton did not specify that any one periodic process should be taken as the measure of Absolute Time. However,



1. Centre of gravity of the solar system taken as the centre of Absolute Space.
2. Selection of the 'best measure' of Absolute Time.
3. Moving bodies construed as systems of indefinitely large numbers of point-masses.
4. Specification of experimental procedures to measure values of impressed forces.

Newton's Interpreted Axiom System for Mechanics

by reading between the lines, one can interpret Newton to have suggested a procedure to link Absolute Time with its sensible measures. Such a link might be established by examining time-dependent sequences which have been determined using various different methods of measuring time. For example, if the distance–time relationship for balls rolled down inclined planes is "more regular" when time is measured by the swings of a pendulum than when time is measured by the weight of water flowing through a hole in a pail, then the pendulum clock is the better "sensible measure" of Absolute Time.[16]

Newton thus carefully distinguished the abstract status of an axiom system from its application to experience. See the diagram above.

Newton enforced the distinction between an axiom system and its application to experience throughout the *Principia*. In the section on fluid dynamics, for example, he distinguished "mathematical dynamics", in which motions are described under various hypothetical resistive conditions, from its application to experience. An application of mathematical dynamics is achieved after experimental determination of how the resistance of a specific medium varies with the velocity of a body moving through it. This distinction between an axiom system and its empirical application was one of Newton's most important contributions to the theory of scientific method. It raised to a new level of sophistication the ideal of the deductive systematization of scientific knowledge.

The third stage of Newton's axiomatic method is the confirmation of the deductive consequences of the empirically interpreted axiom system. Once a procedure is specified to link the terms of the axiom system to phenomena, the investigator must seek to establish agreement between the theorems of the axiom system and the observed motions of bodies.

Newton recognized that the degree of agreement may often be increased by progressive modification of the original assumptions. For instance, he improved the empirical fit of his theory of the moon's motion by modifying the initial assumption that the earth is a homogeneous sphere. This feedback procedure is an important aspect of what I. B. Cohen has termed the "Newtonian Style" in natural philosophy.[17]

Newton himself established extensive agreement between his empirically interpreted axiom system for mechanics and the motions of celestial and terrestrial bodies. An illustration is his experiments with colliding pendulums. Newton showed that after appropriate corrections are made for air resistance, action and reaction are equal regardless of whether the pendulum bobs are composed of steel, glass, cork, or wool.

Newton thus affirmed and practised *two* theories of scientific procedure—the Method of Analysis and Synthesis, and an Axiomatic Method. I think that it does not detract from Newton's genius to point out that he did not keep in mind consistently the distinction between these two theories of procedure.

The Method of Analysis and Synthesis and the Axiomatic Method share as a common objective the explanation and prediction of phenomena. But they differ in an important respect, particularly if one takes a narrow view of what techniques qualify as "induction". The natural philosopher who follows the Method of Analysis seeks to generalize from the results of observation and experiment. The Axiomatic Method, by contrast, places greater emphasis on the creative imagination. The natural philosopher who adopts this method may begin anywhere. But the axiom system he creates is relevant to science only if it can be linked to what can be observed.

"Hypotheses Non Fingo"

Newton agreed with Galileo that primary qualities are the proper subject-matter of physics. According to Newton, the starting-point and end-point of scientific inquiry is the determination of the values of "manifest qualities", those aspects of phenomena that may be measured experimentally.

Newton sought to restrict the content of his "experimental philosophy" to statements about manifest qualities, "theories" derived from these statements, and queries directive of further inquiry. In particular, he sought to exclude "hypotheses" from experimental philosophy.

Newton's use of the terms 'theory' and 'hypothesis' does not conform to modern usage. He applied the term 'theory' to invariant relations among terms designating manifest qualities. He sometimes spoke of these invariant relations as relations "deduced from" phenomena, but he most likely meant by this that there was very strong inductive evidence for certain of these relations. 'Hypotheses', in one of Newton's usages,* are statements about terms that designate "occult qualities" for which no measuring procedures are known.

Newton was quick to take offence whenever his experimentally based "theories" were labelled "hypotheses". For example, when the mathematician Pardies incautiously referred to Newton's theory of colours as a "very ingenious hypothesis",[18] Newton promptly corrected him. Newton emphasized that there was conclusive experimental evidence that sunlight comprises rays of differing colours and refractive properties. He distinguished carefully his "theory" that light has certain properties of refraction, from any "hypothesis" about waves or corpuscles by which these properties might be explained.[19]

Newton defended a similar position on the "theory" of gravitational attraction. He insisted that he had established the existence of gravitational attraction and its mode of operation, thereby accounting for the motions of the planets, the tides, and diverse other phenomena. But he did not wish to jeopardize this "theory" by tying it to a particular hypothesis about the underlying cause of the attraction. "I feign no hypotheses", he wrote.[20]

His injunction was directed primarily against "explanations" of gravitational attraction in terms of the Cartesian hypothesis of invisible swirling vortices of ether. Newton demonstrated in the *Principia* that Descartes's Vortex Hypothesis had consequences that are not in agreement with the observed motions of the planets.

Yet in other contexts, Newton was willing to entertain hypotheses that explain correlations among manifest qualities. Indeed, he himself flirted with a hypothesis about an ethereal medium which produces gravitational attraction. However, Newton emphasized that the function of such hypotheses is to direct future research, and not to serve as premisses for sterile disputation.

The Rules of Reasoning in Philosophy

To direct the search for *fruitful* explanatory hypotheses, Newton suggested four regulative principles, referred to as "hypotheses" in the first edition of the *Principia*, and "rules of reasoning in philosophy" in the second edition. These regulative principles are:

I. We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.
II. Therefore to the same natural effects we must, as far as possible, assign the same causes.
III. The qualities of bodies, which admit neither intensification nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever.
IV. In experimental philosophy we are to look upon propositions inferred by general induction from phenomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions.[21]

In support of Rule I, Newton appealed to a principle of parsimony, declaring that nature "affects not the pomp of superfluous causes". But exactly what Newton meant, or should have meant, by a "true cause" has been a subject of some debate. For instance, both William Whewell and John Stuart Mill criticized Newton for failing to specify criteria for the identification of true causes. Whewell remarked that if Newton meant to restrict the "true cause" of a type of phenomena to causes already known to be effective in producing other types of phenomena, then Rule I would be overly restrictive. It would preclude the introduction of new causes. However, Whewell was not certain that this was Newton's intended meaning. He noted that Newton may have meant only to restrict the introduction of causes to those "similar in kind" to causes that previously have been established. Whewell observed that, thus interpreted, Rule I would be too vague to guide scientific inquiry. Any hypothetical cause could be claimed to display *some* similarity to previously established causes. Having dismissed these inadequate alternatives, Whewell suggested that what Newton should have meant by a "true cause" is a cause represented in

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* I. B. Cohen has discussed nine meanings of 'hypothesis' in Newton's writings (*Franklin and Newton*, 138–40).

a theory, which theory is supported by inductive evidence acquired from analysis of diverse types of phenomena.*

Mill likewise interpreted "true cause" so as to reflect his own philosophical position. Consistent with his view of induction as a theory of proof of causal connection, Mill maintained that what distinguishes a "true cause" is that its connection with the effect ascribed to it be susceptible to proof by independent evidence.†

Commenting on Rule III, Newton indicated that the qualities which satisfy the rule include extension, hardness, impenetrability, mobility, and inertia. Newton maintained that these qualities should be taken to be the universal qualities of all bodies whatsoever. Moreover, he insisted that these also are the qualities of the minute parts of bodies. In Query 31 of the *Opticks*, he set forth a research programme to uncover the forces that govern the interactions of the minute parts of bodies. Newton expressed the hope that the study of short-range forces would achieve an integration of physico-chemical phenomena such as changes of state, solution, and the formation of compounds, in much the same way as the principle of universal gravitation had achieved the integration of terrestrial and celestial dynamics. Subsequently, Newton's research programme received theoretical development from Boscovich and Mossotti, and practical implementation in the electromagnetic researches of Faraday and the various attempts to measure the elective affinities of the chemical elements.‡

The Contingent Nature of Scientific Laws

Newton repudiated the Cartesian programme of deducing scientific laws from indubitable metaphysical principles. And he denied that a necessary knowledge of scientific laws can be achieved in any manner. According to Newton, the natural philosopher may establish that phenomena are related in a certain way, but cannot establish that the relation could not be otherwise.

It is true that Newton did suggest that if we could know the forces that operate on the minute particles of matter, we could understand why macroscopic processes occur in the ways they do. But Newton did not maintain that such knowledge would constitute a necessary knowledge of nature. On the contrary, he held that all interpretations of natural processes are contingent and subject to revision in the light of further evidence.

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* Whewell's concept of a "consilience of inductions" is discussed in Chapter 9.

† Mill's view of causal relation is discussed in Chapter 10.

‡ The role of Newton's research programme in 18th-c. science has been discussed by A. Thackray in *Atoms and Powers* (Cambridge, Mass.: Harvard University Press, 1970).

Notes

[1] Isaac Newton, *Opticks* (New York: Dover Publications, 1952), 404.

[2] Ibid. 45–8.

[3] Newton, *Mathematical Principles of Natural Philosophy*, trans. A. Motte, revised by F. Cajori (Berkeley, Calif.: University of California Press, 1962), ii. 547.

[4] Ibid. i. 8.

[5] Ibid. i. 7–8.

[6] Newton, *Unpublished Scientific Papers of Isaac Newton*, trans. and ed. A. R. Hall and M. B. Hall (Cambridge: Cambridge University Press, 1962), 132–43.

[7] Ronald Laymon, 'Newton's Bucket Experiment', *J. Hist. Phil.* 16 (1978), 399–413.

[8] Newton, *Mathematical Principles*, i. 10–11.

[9] Ernst Mach, *The Science of Mechanics*, trans. T. J. McCormack (La Salle, Ill.: Open Court Publishing Co., 1960), 271–97.

[10] Newton, *Mathematical Principles*, i. 8.

[11] Ibid. i. 13.

[12] Newton, *Opticks*, 154–8.

[13] I. Bernard Cohen, *Franklin and Newton* (Philadelphia: The American Philosophical Society, 1956), 139.

[14] Newton, *Mathematical Principles*, i. 10.

[15] Newton, *Unpublished Scientific Papers*, 127.

[16] e.g. see S. Toulmin, 'Newton on Absolute Space, Time, and Motion', *Phil. Rev.* 68 (1959); E. Nagel, *The Structure of Science* (New York: Harcourt, Brace, and World, 1961), 179–83.

[17] I. Bernard Cohen, *The Newtonian Revolution* (Cambridge: Cambridge University Press, 1980), 52–154.

[18] Ignatius Pardies, 'Some Animadversions on the Theory of Light of Mr. Isaac Newton', in *Isaac Newton's Papers and Letters on Natural Philosophy*, ed. I. B. Cohen (Cambridge, Mass.: Harvard University Press, 1958), 86.

[19] Newton, 'Answer to Pardies', in *Isaac Newton's Papers and Letters on Natural Philosophy*, 106.

[20] Newton, *Mathematical Principles*, ii. 547. See also A. Koyré, *Newtonian Studies* (Cambridge, Mass.: Harvard University Press, 1965), 35–6.

[21] Newton, *Mathematical Principles*, ii. 398–400.